

Extra dimensions as an alternative to Higgs mechanism?

Mikhail Shaposhnikov^{a,1} and Peter Tinyakov^{a,b,2}

^a*Institute of Theoretical Physics, University of Lausanne,
CH-1015 Lausanne, Switzerland*

^b*Institute for Nuclear Research, 117312 Moscow, Russia*

Abstract

We show that a pure gauge theory in higher dimensions may lead to an effective lower-dimensional theory with massive vector field, broken gauge symmetry and no fundamental Higgs boson. The mechanism we propose employs the localization of a vector field on a lower-dimensional defect. No non-zero expectation values of the vector field components along extra dimensions are required. New possibilities for the solution to the gauge hierarchy problem are discussed.

¹E-mail: Mikhail.Shaposhnikov@ipt.unil.ch

²E-mail: Peter.Tinyakov@cern.ch

It is usually believed that fundamental or composite scalar fields are necessary for construction of self-consistent theory of interacting massive vector bosons. Indeed, the only known renormalizable theories of massive vector bosons in 4d are gauge theories with spontaneous symmetry breaking by the Higgs mechanism, while all theories that have good high energy behaviour of tree amplitudes can be shown [1] to be equivalent to the spontaneously broken gauge theories with scalar fields.

In this note we show that a higher-dimensional pure gauge theory may lead to an effective low-energy theory of *massive* vector bosons in lower dimensions, thus breaking the gauge symmetry without fundamental Higgs boson. Our mechanism makes use of the localization of a vector field on a lower-dimensional defect (for localization of massless vector fields on a brane see [2] - [8]). It does not involve scalar fields and does not assume non-zero expectation values of the vector-field components along extra dimensions. It is also quite different from the Hosotani mechanism [9] related to a non-trivial topology of the compact extra dimensions.

In the usual Kaluza-Klein (KK) compactification, the low-energy sector consists of the fields which are constant along the compact dimensions and correspond to the lowest KK mode. Each higher-dimensional vector field produces a massless vector and massless scalars (the vector components in the direction of the compact dimensions) in the low-dimensional effective theory. Higher KK modes give massive fields. The gauge symmetry of the higher-dimensional theory with respect to gauge transformations which do not depend on the compact dimensions translates into the gauge symmetry of the effective theory. The massive KK fields are invariant under this symmetry.

The mechanism we propose leads to a theory which is similar in many respects. The crucial difference is that the analog of the massless KK sector disappears from the theory, while massive sectors are organized in such a way that one is much lighter than the others. As a result, the effective low-energy theory contains only a massive vector field and has no gauge symmetry.

Consider for definiteness the following gauge-invariant Lagrangian in 5d (generalizations to higher dimensions are obvious):

$$S = -\frac{1}{4} \int d^4x dz \Delta(z) F_{AB} F^{AB} , \quad (1)$$

where F_{AB} is the ordinary field strength and $\Delta(z) > 0$ is some weight function depending, in general, on the fifth coordinate z . The origin of the weight function is not essential in what follows. For instance, it may arise in a natural way from warped compactifications on 3-branes [10, 11, 12]. For simplicity, we assume that $\Delta(z)$ is an even function of z . In the case $\Delta(z) = 1$ we have an ordinary 5d action. We are going to show that, depending on the behaviour of $\Delta(z)$ at infinity, the low-energy effective theory may describe massive 4-dimensional vector bosons without any scalar fields.

Let us consider first an Abelian gauge field. One can expand the field $A_B(x_\nu, z)$ in a Fourier-type series with respect to the fifth coordinate,

$$A_B(x_\nu, z) = \sum_n A_B^n(x_\nu) \psi_n(z). \quad (2)$$

From equations of motion for the gauge field it follows that the functions $\psi_n(z)$ obey the equation

$$-\frac{1}{\Delta(z)}\frac{\partial}{\partial z}\left(\Delta(z)\frac{\partial}{\partial z}\psi_n(z)\right)=m_n^2\psi_n(z) \quad (3)$$

and the following orthogonality and completeness conditions:

$$\int dz\Delta(z)\psi_n(z)\psi_m(z)=\delta_{mn}, \quad \sum_n\psi_n(z)\psi_n(z')=\frac{1}{\Delta(z)}\delta(z-z'). \quad (4)$$

From a 4-dimensional point of view, $A_\mu^n(x_\nu)$ are vector fields, while $A_z^n(x_\nu)$ are scalars (here and below the Greek indices run from 0 to 3 and correspond to 4-dimensional space-time).

The eigenvalues m_n^2 determine masses of the fields. As follows from eq.(3) upon multiplication by $\Delta(z)\psi_n(z)$ and integration over z , they are all non-negative. In the gauge $A_z = 0$ the Lagrangian reads

$$L = \sum_n \left(-\frac{1}{4}F_{\mu\nu}^n F_n^{\mu\nu} + \frac{1}{2}m_n^2(A_\mu^n)^2 \right), \quad (5)$$

where

$$F_{\mu\nu}^n = \partial_\mu A_\nu^n - \partial_\nu A_\mu^n. \quad (6)$$

Note that at $m_0^2 \neq 0$, this Lagrangian describes a collection of massive vector fields, while the scalar components have disappeared from the spectrum. The counting of number of degrees of freedom works in a trivial way: massless vector field in 5d has the same number of physical degrees of freedom as the massive vector field in 4d.

If the fields corresponding to $n > 0$ are much heavier, $m_n^2 \gg m_0^2$, one may expect that they decouple and eq.(5) truncated at $n = 0$ is the correct effective Lagrangian at low energies. Thus, starting from the gauge-invariant 5-dimensional theory (1) we arrive at the 4-dimensional effective theory (5) containing a massive vector field and no scalars.

Let us now show that the desired situation — non-zero m_0^2 separated by a gap from the rest of the eigenvalues — occurs for a wide class of weight functions $\Delta(z)$. The first thing to note is that eq.(3) always has a solution $\psi(z) = \text{const}$ at zero m^2 . If the integral $\int dz\Delta(z)$ is convergent, the low energy effective theory contains massless 4-dimensional vector field (in other words, the gauge field is localized in 4 dimensions). This is analogous to the gravitational localization in the case of a local string-like defect in 6d [13, 3, 4, 5] or a domain wall in 5d with a dilaton field [7]. Weight functions of that type could be used to describe unbroken gauge symmetry on the brane. Depending on the specific form of $\Delta(z)$, the theory may contain a continuum of states living in the bulk without a mass gap, as in the cases of refs.[4, 5, 7]. For other choices of the weight function the spectrum of states may develop a mass gap or be discrete. For example, for $\Delta(z) = \exp(-M|z|)$ with $M > 0$ the zero mode is separated from continuum by a mass gap M , whereas for $\Delta(z) = \exp(-M^2 z^2)$ the spectrum of states is discrete with $m_n^2 \sim M^2 n$.

If the integral $\int dz\Delta(z)$ diverges, the solution $\psi(z) = \text{const}$ is non-normalizable. The linearly independent solution to eq.(3) with $m^2 = 0$ has the explicit form $\psi(z) = \int^z dx \Delta^{-1}(x)$. Neither the second solution, nor any linear combination with the first one is normalizable. Thus, in

this case zero mode does not exist and m_0^2 is positive. This is the situation we are interested in.

If the weight function $\Delta(z)$ decreases at small z , reaches a minimum and then grows fast enough at large z (see Fig. 1a), the lowest eigenvalue m_0^2 is generically small. In order to see this we first rewrite eq.(3) in the Schrödinger form by changing the variables according to $\chi = \Delta^{1/2}\psi$,

$$\left(-\frac{d^2}{dz^2} + V(z)\right)\chi_n(z) = m_n^2\chi_n(z) \quad (7)$$

with the potential

$$V(z) = W^2 - W', \quad W = -\frac{\Delta'}{2\Delta}. \quad (8)$$

For the weight function of Fig. 1a the potential is shown in Fig. 1b; it has a well at small z which gives rise to the lowest level with almost zero eigenvalue. If $\Delta(z)$ grows exponentially or faster at infinity, $V(z)$ goes to a constant or grows as well, and the spectrum has a gap or even is discrete.

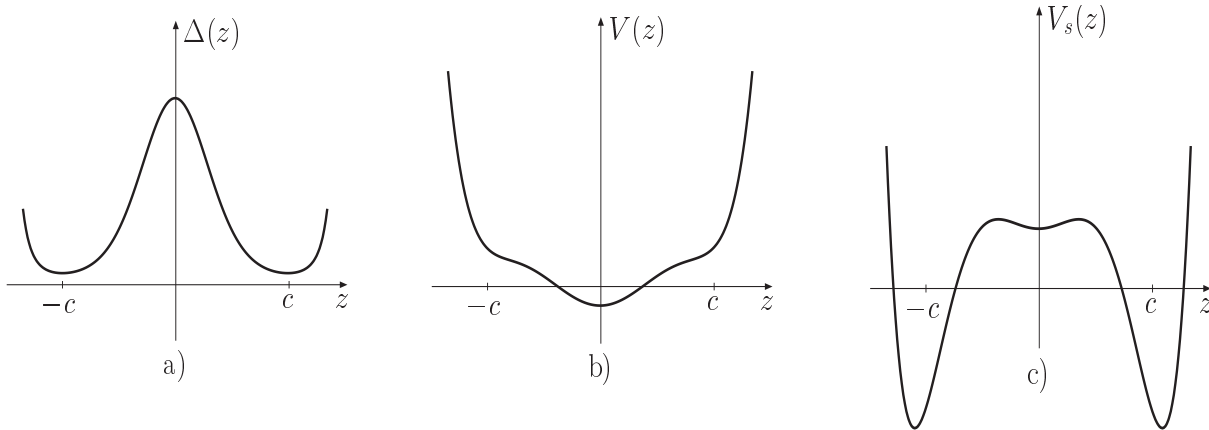


Figure 1: The weight function $\Delta(z)$, corresponding quantum-mechanical potential $V(z)$ and its SUSY-partner potential $V_s(z)$.

We now note that the potential (8) has the same form as in the supersymmetric quantum mechanics (for a review see [14]). Thus, one can immediately construct a SUSY-partner potential, $V_s(z) = W^2 + W'$, which has the same discrete levels plus an extra one, the exact zero mode [14]. The lowest level of the original potential (8) is the first excited state for the SUSY-partner potential V_s . The SUSY-partner potential corresponding to the weight function of Fig. 1a is plotted in Fig. 1c. It is of the double-well type, the wells corresponding to the minima of $\Delta(z)$. The levels which lie below the barrier separating the two minima come in pairs with exponentially small splitting. Since we know that the lowest eigenvalue is exactly zero, the next one (which is, in turn, the lowest for the original potential (8)) is exponentially small. This smallness is due to the exponential suppression of the tunneling between the two wells.

As a first specific example consider the weight function of the form

$$\Delta(z) = \exp\left(-M|z| + \frac{1}{2}m^2 z^2\right), \quad M > 0, \quad m^2 > 0, \quad (9)$$

which corresponds to the potential and SUSY-partner potential

$$\begin{aligned} V(z) &= \frac{1}{4} \left(M - m^2|z|\right)^2 + \frac{1}{2}m^2 - M\delta(z), \\ V_s(z) &= \frac{1}{4} \left(M - m^2|z|\right)^2 - \frac{1}{2}m^2 + M\delta(z). \end{aligned}$$

The lowest level m_0^2 is zero to all orders of perturbation theory in m^2/M^2 . The simplest way to see this is to note that this perturbation theory would correspond to expanding the weight function to some finite order in m^2 ,

$$\Delta(z) = \exp(-M|z|) \left(1 + \frac{1}{2}m^2 z^2 + \dots\right).$$

Since the resulting weight function exponentially decreases at large z , eq.(3) has an exact normalizable zero mode $\psi_0 = \text{const}$ no matter at which order the expansion is truncated. Thus, perturbative corrections are equal to zero. The semiclassical calculation of the splitting of the two lowest levels in the SUSY-partner potential $V_s(z)$ gives, for $M^2 \gg m^2$,

$$m_0^2 \sim Mm \exp\left(-\frac{M^2}{2m^2}\right),$$

while higher levels are discrete and have masses of order m and higher.

Another example with a smooth weight function is

$$\Delta(z) = \exp\left(-\frac{1}{2}M^2 z^2 + \frac{1}{4}m^4 z^4\right), \quad M^2 > 0, \quad m^4 > 0. \quad (10)$$

The corresponding potential and SUSY-partner potential are

$$\begin{aligned} V(z) &= \frac{1}{4}z^2 \left(M^2 - m^4 z^2\right)^2 - \frac{1}{2}M^2 + \frac{3}{2}m^4 z^2, \\ V_s(z) &= \frac{1}{4}z^2 \left(M^2 - m^4 z^2\right)^2 + \frac{1}{2}M^2 - \frac{3}{2}m^4 z^2. \end{aligned}$$

In this case the lowest eigenvalue is, for $M^2 \gg m^2$,

$$m_0^2 \sim \frac{M^4}{m^2} \exp\left(-\frac{M^4}{4m^4}\right). \quad (11)$$

The higher levels are also discrete and have masses starting from M .

In both examples, the lowest eigenvalue is determined, up to a numerical coefficient, by the ratio $\Delta(c)/\Delta(0)$, where $\Delta(c)$ is the minimum of the weight function (see Fig.1a). This is not

a coincidence. Indeed, if $W' \ll W^2$, as in the above examples, the leading contribution to the tunneling exponent

$$m_0^2 \propto \exp \left(-2 \int_0^{z_*} \sqrt{V_s(z)} dz \right) \sim \exp \left(-2 \int_0^{z_*} W(z) dz \right) ,$$

where z_* is a root of the equation $V_s(z) = 0$, is

$$m_0^2 \propto \frac{\Delta(c)}{\Delta(0)}.$$

Thus, the exponential smallness of the lowest energy level in the above examples is related to the exponential form of the weight function. It may lead to a solution of the gauge hierarchy problem, provided the weight function of exponential form arises naturally.

Consider now in more detail the effective theory for light modes in the case of a non-abelian gauge field in 5 dimensions. This is a 4-dimensional theory of massive vector field with the standard cubic and quartic interactions

$$g_3 f^{abc} \partial_\mu A_\nu^a A^{b\mu} A^{c\nu} + \frac{g_4}{4} f^{abc} f^{adf} A_\mu^b A_\nu^c A^{d\mu} A^{f\nu} ,$$

where the couplings are expressed in terms of the 5-dimensional gauge coupling G by the following equations,

$$g_3 = G \int dz \Delta(z) \psi_0^3(z), \quad (12)$$

$$g_4 = G^2 \int dz \Delta(z) \psi_0^4(z). \quad (13)$$

Two cases should be distinguished. In the first one the integral $\int dz \Delta(z)$ converges, and the theory has a normalizable zero mode, which does not depend on z . In this case an effective 4-dimensional theory is an unbroken gauge theory with $g_4 = g_3^2$, as is dictated by the 4-dimensional gauge invariance.

In the opposite case the gauge symmetry in 4d is realized in a different way. To understand it better we put the system in a finite box with $-z_0 < z < z_0$, and consider a limit $z_0 \rightarrow \infty$ at the end. Then, the constant wave function $\bar{\psi}_0 = \text{const}$ (we put a bar to distinguish it from the lowest massive mode) is normalizable and is a part of the physical spectrum. As in the discussion above, it is orthogonal to other modes. The corresponding 4d gauge field \bar{A}_μ^0 has a zero mass and the effective Lagrangian has 4d gauge invariance, with the massive fields being the gauge singlets. Now, if $\Delta \rightarrow \infty$ at $z \rightarrow \infty$, one can easily see that this massless vector field decouples from the other fields in the limit $z_0 \rightarrow \infty$ and, therefore, is not observable. Indeed, the gauge coupling of this field is simply

$$\bar{g} = G \frac{\int dz \Delta \bar{\psi}_0^3}{\int dz \Delta \bar{\psi}_0^2} = \frac{G}{\sqrt{\int dz \Delta}} \rightarrow 0 \quad (14)$$

when $z_0 \rightarrow \infty$.

As for the self-interaction of the lowest massive mode, it survives in the limit $z_0 \rightarrow \infty$, but the relation between quartic and cubic couplings may acquire corrections. By the same argument as in the case of m_0^2 , the ratio g_3^2/g_4 cannot have perturbative corrections. Thus, in the case of the weight function (9) one expects

$$g_4 = g_3^2 \left(1 + O \left(\frac{\Delta(c)}{\Delta(0)} \right) \right).$$

A few comments are now in order:

(i) If the initial theory is formulated in five or higher dimensions, it is not renormalizable, so it is not surprising that the low-energy theory is not renormalizable either. In this case one may hope that if the higher-dimensional theory is in turn an effective field theory coming from string theory, the divergences will be cut at higher energy scale. Moreover, the effective theory is nothing but a non-linear gauged sigma-model [15, 16]. This model is known to be strongly interacting at the energy scales of the order $(64\pi m_0^2)/3g_4$ [17].

(ii) An interesting situation arises if we start with a gauge theory formulated in the space-time of 4 or lower dimensions. In this case the initial theory is renormalizable, but the low-energy theory in the space of lower dimensions is not. Divergences disappear after summing up all heavy degrees of freedom!

(iii) If the electroweak symmetry breaking is to be attributed to this mechanism, the $\gamma - Z$ mixing must be explained. In the simple examples we considered above all vector bosons of a simple gauge group acquire the same masses, and, since the weight function was a gauge singlet, the required mixing cannot appear at all. So, to get a phenomenologically acceptable situation, the effective weight function should carry gauge indices as well. Potentially, that could happen in Kaluza-Klein type compactifications [18] generalized to the warped metric case, or to non-trivial gauge backgrounds [19, 20].

(iv) The effective low energy action discussed above does not include any radiative corrections. It remains to be seen whether they affect the 4-dimensional character of the effective theory.

In conclusion, we proposed that higher dimensional gauge theories may lead to low energy theories for massive vector bosons without fundamental scalar fields. It would be interesting to see whether the weight factors required can arise in a natural way from warped compactifications on topological defects or in warped string compactifications [21]¹.

We wish to thank S. Dubovsky, T. Gherghetta, M. Laine and V.A. Rubakov for helpful discussions. This work was supported by the FNRS, contracts no. 21-55560.98, 7SUPJ62239 and 21-58947.99.

¹After our paper appeared in arXiv, a paper, [22], appeared in which spontaneous symmetry breaking in higher dimensional theories was addressed in a different manner.

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